COMBINED MATHEMATICS

By PROFESSOR W. PAUL WEBBER
University of Pittsburg

The writer has found a good many teachers of high school mathematics who are not clear as to the nature and significance of combined mathematics, or as some term it, fused mathematics. He has further found that a considerable number of these teachers are honestly, though not always very actively, seeking light on the subject. In localities where the adoption of such courses in high schools is imminent, the question is one of considerable interest. In talking with teachers there has been found a wide divergence of opinion as to what constitutes a combined course in mathematics. Even the text books do not appear to convey a uniform opinion on this point. In what follows, the writer, in giving his views, may inadvertently appear to take sides.

It may be well to consider first, very briefly, the origin of combined mathematics from the pedagogical standpoint. How has combined mathematics come into existence? A number of the best and most conscientious teachers recognized the unsuitableness of the traditional courses under existing conditions in school. They at last recognized that Euclid and his successors were not suited to juvenile instruction. Euclid was originally intended for mature adults. A similar conclusion was reached regarding the traditional algebra. These teachers set about constructive remedial measures. They were patient and devoted, and finally became insistent. Combind mathematics is one of the results of their efforts. In so far as their testimony goes, considering their reputation as teachers, it must command serious attention. They had no idea of elimination. They wished to make the courses in mathematics more atractive and more efficient as an educational instrument. No claim of perfection or finality comes from them. We, as teachers, are to try out, and to further improve what they have given us.

One of the greatest difficulties to an understanding of combined mathematics is lack of information. There is, also, the ever present difficulty in the way of every new move, fear. This is psychological and will vanish when the first has been re-

moved. To some, the term combined mathematics, seems to suggest the impossible task of teaching simultaneously, arithmetic, algebra and geometry in some form not very dissimilar to the present traditional courses. To others, the term suggests an undefined "hotch potch." To still another group, the term suggests a sort of materialistic substitute which is not mathematics at all. So it goes on. Now, as the writer conceives combined mathematics, these are all wrong or at least, very misleading.

Taking up the first difficulty mentioned above, it is to be observed that the traditional mathematical curriculum of a standard high school is usually: one year of algebra, one year of geometry, half a year or a year of advanced algebra, followed by solid geometry or plane trigonometry, or both. Plainly, this course is a succession of partial courses or branches taken through the high school period. That is, we have an alternation of topics in rather large units of years and half years. One thing that combined mathematic purposes is to alternate the topics in smaller units, very much smaller units. Thus the difficulty of simultaneity as above indicated is removed.

Then comes the question of the unifying or co-ordinating principle which is to hold the course together. Let us first ask what has been the unifying principle of the traditional courses. In geometry, it has been logical sequence for the most part, and a fair degree of completeness as a final goal. In algebra, it seems to have been the historical order of development of the operations of arithmetic with the generalized numbers, as is the fashion in ordinary arithmetic. This may be termed a cumulative instrumental value of processes. Individually, each process is developed more or less logically. Now what appears to be the unifying principle of combined mathematics? In answer we may say there are several. The author of one text on combined mathematics for college freshmen has used the notion of function as a connecting principle. In fact, each chapter is roughly a more or less complete mathematical treatment of a single type or group of functions. Various operations are applied, in order, to each type. The types increase in complexity as the course proceeds. Another author has apparently taken the cumulative instrumental value as a unifying principle. There is evidence that he has made use of the function idea in

certain parts. Other writers have made use of these and other principles in constructing courses in combined mathematics. Only the most outstanding principles have been pointed out. For a complete analysis would show various subordinate principles woven into the larger plan. To mention all these would take us beyond our limits in this paper.

It seems to the writer, that the function idea, in any of its current forms, would not be well adapted as a unifying principle in courses for first year high school pupils. The principle of cumulative instrumental value seems practicable and more in line with practice in some of the older courses.

It may now be asked whether the logical consistency of the development need be sacrificed when some principle, other than logical sequence, is made fundamental in binding the course together. The answer is certainly in the negative. For there is ample place for the principle of logical sequence as well as scientific proceedure in the development of particular topics. This will be illustrated later. Then what is to be gained by changing from the time honored course to the combined course? Different advocates will offer different claims. To the writer, some advantages stand out clearly.

(1) It is possible to so arrange the course in a sequence that the most useful (both mathematically and practically) can be presented in cumulative order, while the acknowledged difficult parts can be postponed to a later stage, when they will be more easily approached.

(2) By bringing algebraic methods and geometric methods into close relation from the start, a longer period of training in correlating these methods is gained.

(3) For students who will not go far in mathematics, either on account of taste or ability, the combined course seems to offer more that is useful (or usable) and within the grasp of the weaker (mathematically) than is possible under the old plan.

(4) Those who may desire to take fore advanced courses will approach them with a better perspective of their content and purpose. As a consequence, they will make more rapid progress in the more specialized branches, such as formal algebra and demonstrative geometry.

(5) Saving of time has been offered as a distint advantage. To the writer it seems this may be open to question. But teachers who have tried it claim that both plane and solid geometry can be done satisfactorily in a single year, following a good course in combined mathematics. They also claim that formal algebra can be covered through "third semester algebra" in a single year, after combined mathematics. This appears to be entirely feasible. It appears that in three years, pupils beginning with a year of combined mathematics can get as far in formal mathematics as under the old plan in the same time. They will probably have a much better use of their mathematics at that stage under the new plan. It remains to be seen whether there can be an actual material shortening of the time to cover three years' work. The fact remains that no time is lost by the new plan.

It may be suggestive to outline an example of proceedure for the sake of definiteness. For example, one book starts out with the use of the simple equation as an instrument in solving concrete problems within the understanding of the pupils. Incidentally, but with definite purpose, through the first chapter some of the terms and fundamental processes relating to equations are used and explained, such as:

1. Equal numbers may be added to both members of an equation.

2. If both members of an equation be multiplied by the same or equal numbers, the results will be equal.

In the next chapter, some definite fundamental geometric facts and principles are introduced. Then the equation is applied to a new set of problems involving the geometric knowledge just gained.

In a following chapter, some further algebraic and geometric principles are developed and applied to problems of a more advanced type. In general, the entire year's work is developed systematically and with as much logical soundness as the pupils are capable of appreciating. There is thus accumulated a stock of intuitional and experimental geometry correlated with a working knowledge of the fundamentals of algebra in the use of simple forms. Such a foundation will, with little doubt, form

a good basis for the study of a standard course in formal algebra or geometry.

It is to be remembered that the claims above mentioned are dependent upon good teaching. It follows that a responsibility rests with teachers for the success of the plan. It must be remembered that the old plan has been discredited, rightly or wrongly, and cannot be restored if the new plan should fail. The question comes, "What then?" The answer is, *no required mathematics* in the high school curriculum. Such a result would be little short of a catastrophe in education, and while it probably could not be permanent, it would take a generation to impress the fact upon the public, and that much time would be lost. The logical thing to do is to make the revised plan a success as far as possible.

Many teachers object to combined mathematics on the ground that it is not sufficiently thorough. What is thoroughness? Is it not possible that some confuse thoroughness with completeness, in the sense of covering all that is available in a subject? Thoroughness in elementary instruction cannot be measured by completeness. It must be measured by the degree of fixedness of that which is attempted. A course in algebra covering only the simple equation is, so far as concerns the solution of concrete problems, or may be, thorough. This much is done thoroughly when pupils can use such knowledge well and readily. Such a course would not be a complete course in algebra, however. We will say, then, that a course is thorough if it trains pupils well in the use of a set of methods and principles, even if limited to simple matters. The thoroughness here advocated lies more in training in the use of scientific proceedure. It is believed that herein lies the educational value of mathematics, if there be any beyond the few facts needed in daily life.

In closing, a brief outline of a method of scientific proceedure will be given. A similar one has been given during the last several years by the writer to his classes in the "Theory and Methods of Teaching Secondary Mathematics."

Reasoning may be defined as a rearranging and a recombining of ideas and facts with a definite purpose. In all reasoning it is necessary to have a definite starting point, to understand the meaning of all terms used in the given data and in the processes

of rearranging the ideas. It is further necessary to have definite rules, or criteria of proceedure. To be specific:

1. The given data, conditions or hypotheses must be clearly known.

2. The objective or conclusion (thing to be done) must be known.

3. Every term used must be defined (understood).

4. There must be an analysis of the given conditions and of the objective, with the view to finding a set of facts, ideas or steps leading from the data to the objective.

5. Assemble all facts and ideas that are known and that seem to be related to the problem.

6. Select a succession of steps or relations that will lead in an orderly, convincing way from the data to the objective.

7. When a selection of steps has been found and tried, and found unsatisfactory, another set must be sought and tried out.

8. Having found a set of steps that is satisfactory, state the result and interpret it, indicating any special cases or restrictions that are to be observed.

The question arises here as to whether we have given as much effort to teaching scientific methods of proceedure as to teaching mathematical facts. Should not every new problem be made a means of cultivating the habit of scientific methods of proceedure?

Without burdening this paper with further details, the writer can hardly do better than to refer the reader to ***Fundamental Aspects of Mathematical Training***, by Prof. Sanders, Bulletin of Louisiana State University, Vol. XIII-N. S., February, 1921, where an excellent collection of carefully worked out illustrations may be found.